

GENERATION OF AVAILABLE POTENTIAL ENERGY AND THE INTENSITY OF THE GENERAL CIRCULATION

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I. The Energy Cycle and the Intensity of the General Circulation

IN my talk today, I shall attempt to account for the intensity of the general circulation through a consideration of the physical processes that maintain the atmospheric energy cycle. The energy cycle may be regarded as consisting of a net generation of potential plus internal energy by non-adiabatic effects, a net conversion of potential plus internal energy to kinetic energy by adiabatic processes and a net dissipation of kinetic energy by viscosity (which converts the kinetic energy back into potential plus internal energy) as shown schematically in Fig. 1. On the average over a long period of time, the individual links in the energy cycle (represented by the arrows in Fig. 1) must have the same magnitude.

One measure of the intensity of the general circulation is the amount of

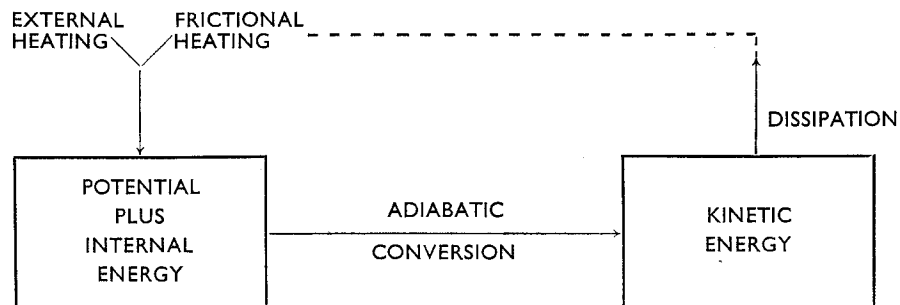


FIG. 1. Schematic picture of the atmospheric energy cycle.

kinetic energy contained in the atmosphere. Since the rate of dissipation of this kinetic energy is roughly proportional to the kinetic energy itself, the rate of dissipation (and hence also the other links in the energy cycle) may also be regarded as a measure of the intensity of the general circulation. Brunt (1944) has estimated this rate to be of the order of 0.02 of the incoming solar radiation. Other estimates, while different, are of the same order of magnitude.

It would be of value to secure an independent estimate of one of the other links in the energy cycle and to compare this with the estimated rate of dissipation. If we attempt to determine the rate of generation of potential plus internal energy by non-adiabatic effects, however, we are faced with the inevitable result that the total generation over a long period of time is equal to the generation by frictional heating alone, since the net heating of the atmosphere over a long period of time by non-frictional processes (i.e. the external heating) is zero. But, the rate of heating by friction is identically equal to the rate of dissipation. Hence, no estimate of the total heating can be independent of the estimated rate of dissipation. We might, instead, examine the net rate of conversion from potential plus internal energy to kinetic energy by adiabatic processes. This conversion depends on the distribution of vertical motion in the atmosphere. Since vertical motion is too small to be measured directly, it would be necessary to infer it indirectly with the aid of the dynamical equations. A legitimate question might be raised as to whether such inferences provide an *independent* measure of the conversion process.

One method of circumventing these difficulties is to express the energy cycle in terms of 'available potential energy', instead of total potential plus internal energy. Available potential energy is a measure of the maximum amount of potential plus internal energy that is available for conversion (by adiabatic processes) to kinetic energy. The instantaneous rate of generation of this form of energy is not necessarily the same as the rate of generation of total potential plus internal energy. The following considerations serve to illustrate the differences:

Let us consider a fluid which is horizontally stratified. The fluid might possess a great deal of potential plus internal energy, but, if the stratification is stable, none of this energy is available for conversion to kinetic energy. If, now, we heat the fluid at one point, a horizontal pressure gradient will be created and a certain amount of potential plus internal energy will become available for conversion to kinetic energy. In this case, both available potential energy and total potential plus internal energy are increased. If, however, we cool the fluid at one point, instead of heating it, a horizontal pressure gradient will again be created and a certain amount of potential plus internal energy will again become available for conversion to kinetic energy. In this case, the total potential plus internal energy is *decreased*, while the available potential energy is *increased*. It is obvious, therefore, that the rate

of generation (G_0) of total potential plus internal energy by non-adiabatic processes is different from the rate of generation (G) of available potential energy by non-adiabatic processes. The former may be expressed in the form

$$G_0 = \int Q \, dM, \quad (1)$$

and the latter in the form

$$G = \int NQ \, dM, \quad (2)$$

(see Lorenz, 1955a), where Q is the net rate of heating of the atmosphere per unit mass, M is mass, $N = 1 - p^{-\kappa} \bar{p}^{\kappa}$, p is pressure, $\kappa = 1 - \frac{c_v}{c_p}$, c_v and c_p are

the specific heats of dry air at constant volume and pressure, respectively, and \bar{p} is a quantity whose value at any point is equal to the average value of p over the isentropic surface which passes through that point. G is seen to depend on a correlation between N and Q over the mass of the atmosphere. It can be shown (Lorenz, 1955b) that within a pressure surface N is a monotone increasing function of the temperature.

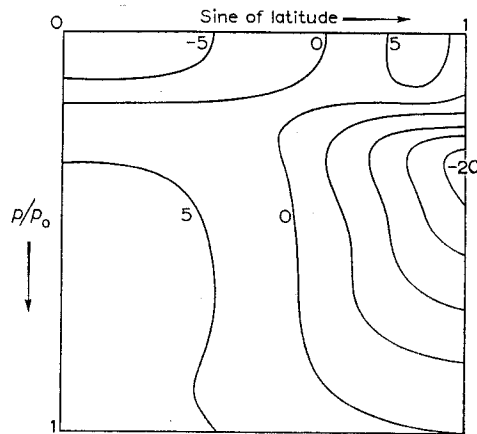


FIG. 2. Distribution of N averaged over longitude and time, based on average temperatures for the four months January, February, July and August 1949, as tabulated by Mintz (1955).

Fig. 2 shows the distribution of N in the atmosphere, averaged over longitude and time. This distribution is such that there must be a large positive correlation between N and the net radiational heating of the atmosphere. On the basis of our present knowledge of the distribution and magnitude of frictional dissipation, however, it is safe to say that the contribution to G due to a correlation between N and the frictional heating is negligible. Thus, although G_0 depends entirely on frictional heating, G depends almost exclusively on non-frictional heating.

Now, since the rate of conversion between *total* potential plus internal energy and kinetic energy is the same as the rate of conversion between *available* potential energy and kinetic energy, the energy cycle may be regarded as consisting of a net generation of available potential energy by non-adiabatic processes, a net conversion of available potential energy to kinetic energy, and a net dissipation of kinetic energy by friction. Furthermore, since frictional dissipation has a negligible feedback into the generation of available potential energy by non-adiabatic processes, it is possible to obtain an estimate of G which is independent of any estimates of the dissipation. Such an estimate, based on averages of N and Q over longitude and time, yields a value of about 2 per cent of the incoming solar radiation, thus agreeing well with Brunt's estimate of frictional dissipation.

II. The Maximum Possible Intensity of the General Circulation

We might then ask why the efficiency of the atmosphere is as low as it is. In order to gain some insight into this problem, we shall attempt to determine the maximum possible rate of generation of available potential energy that can take place in the atmosphere with the existing distribution of incoming solar radiation. To begin with, we may note that if there were no horizontal temperature contrast, there would be no generation of available potential energy in spite of the intense heating and cooling that would prevail. At the other extreme, if the atmosphere were everywhere in radiative equilibrium, there would again be no generation of available potential energy in spite of the large temperature contrast that would prevail. If, however, the horizontal temperature contrast were intermediate between these two extremes, the contrast in the heating would also be intermediate, and the temperature (and hence the value of N) would be positively correlated with the heating. There would thus be a positive generation of available potential energy. Among the different temperature distributions in the last category, there may be one which leads to the maximum possible rate of generation of available potential energy. In principle, this maximum rate can be determined. In practice, the problem is complicated by the fact that the distribution of heating depends not only on the incoming solar radiation and the temperature field, but also on the distribution of clouds, water vapour and ozone.

For simplicity, calculations have been made for a hypothetical atmosphere containing a single absorbing constituent whose absorption coefficient is independent of wavelength. This constituent is assumed to have a constant mixing ratio. It is assumed, too, that all solar radiation which is not reflected back to space is absorbed by the ground. The ground radiates as a black body. All heat transfer between the ground and the atmosphere, and within the atmosphere, is accomplished by long-wave radiation. Friction is distributed so as not to generate any available potential energy.

The model is therefore based on the assumption that the radiational heating, and hence the integral $\int NQ \, dM$, is a function of the temperature field and the incoming solar radiation. If the solar radiation is assumed known, the temperature field which maximizes this integral, and the maximum value of the integral, are found by the method of calculus of variations subject to the constraint that $\int Q \, dM = 0$ (i.e. that the net incoming and outgoing radiation must balance). Ignoring variations with height and with longitude, and assuming a reasonable value for the mean albedo, we find that the maximum possible rate of generation of available potential energy in such an atmosphere is about 0.01 of the incoming solar radiation.

This is less than the estimated value of the actual rate of conversion in the real atmosphere. If the estimate for the real atmosphere is correct, it follows that we have failed to take some factor into account. The discrepancy could arise from the neglect of the vertical dimension, or from the neglect of *variations* of the albedo. In order to determine the effect of including the vertical dimension, we are currently making similar calculations for a two-dimensional model. Preliminary results suggest that the maximum possible rate of generation of available potential energy lies somewhere between 0.01 and 0.04 of the incoming solar radiation. As for the variation in the albedo, this can be taken into account in the one dimensional model. Using reasonable values for this variation, we find that the maximum possible rate of generation of available potential energy is increased by 50 per cent of its value since the albedo is smaller at low latitudes and greater at high latitudes. The main conclusion that we derive from the present results is that the general circulation of the atmosphere may be operating near its maximum possible rate.

We may digress for a moment to note that time variations in the albedo offer a possible mechanism for irregular fluctuations in the intensity of the general circulation (i.e. for an index cycle). Suppose, for example, that the general circulation is operating at about maximum intensity, and that for some reason (e.g. the disappearance of snow, or an equatorward shift of the belt of maximum cloudiness) the albedo at high latitudes decreases relative to that at low latitudes so that the variance of the heating is reduced. In this case, the maximum possible intensity of the general circulation would decrease and available potential energy could no longer be generated at the rate at which kinetic energy is being dissipated. The amount of kinetic energy in the atmosphere must then decrease. Suppose that after awhile the albedo returns to its original distribution. The maximum possible intensity of the general circulation will then increase. It seems plausible, too, that the actual rate of generation of available potential energy would ultimately increase also, and that an increase of kinetic energy would follow, thus completing the index cycle.

III. Maintenance of the General Circulation Near its Maximum Possible Intensity

It is now appropriate to ask whether there is any reason why the actual intensity of the general circulation should be near its maximum possible value. A possible answer to this question is suggested by the results of the Hide annulus experiment described earlier by Fultz. Like the atmosphere, the fluid in the experiment possesses an energy cycle. Let us consider the intensity of this cycle in the case of the steady symmetrical regime. In order that G be different from zero, and in order that a steady state be maintained, the horizontal temperature contrast (ΔT) must lie between zero and the value

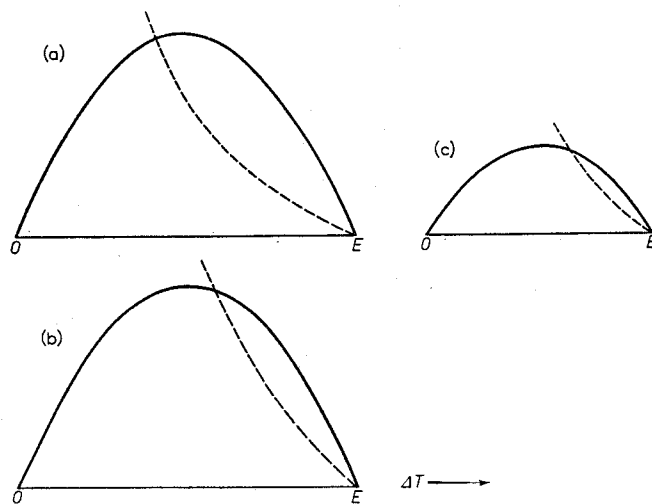


FIG. 3. Schematic diagram showing the variation of G (solid curve) and D (dashed curve) with ΔT . (a) Moderate rotation and moderate heating. (b) High rotation and moderate heating. (c) Moderate rotation and weak heating. E is the value of ΔT at thermal equilibrium.

corresponding to thermal equilibrium (E). The solid curve in Fig. 3 shows schematically the variation of G with ΔT . In order that ΔT be less than E , there must be a large-scale exchange process. In the symmetrical case, a certain minimum meridional circulation is required to accomplish this exchange. The meridional circulation itself contains comparatively little kinetic energy, but the action of the Coriolis force on the circulation generates an appreciable zonal flow. The zonal flow, in turn, is counteracted by frictional dissipation (D). It follows that a value of ΔT less than E implies a certain minimum value of D . The minimum value of D as a function of ΔT is shown schematically by the dashed curve in Fig. 3.

The existing ΔT and the corresponding G must lie to the right of the intersection of the solid and dashed curves in Fig. 3. In Fig. 3a, this intersection lies to the left of the highest point on the curve of G so that the general circulation

may operate at its maximum rate. If, however, the rate of rotation is increased, the same ΔT requires the same minimum meridional circulation, but this circulation requires a more intense zonal circulation, and hence a greater value of D . The dashed curve in Fig. 3 is then raised. If it is raised sufficiently, it will intersect the solid curve to the right of its highest point, and the general circulation cannot proceed at its maximum rate, as shown in Fig. 3b. If, instead, the heating contrast (represented by E) is reduced, the solid curve is lowered, but the dashed curve is not lowered relative to E as an origin. Thus, the dashed curve may again intersect the solid curve to the right of its highest point, as shown in Fig. 3c.

It follows that if the rotation rate is too great, or the heating contrast too small, the general circulation of the fluid in the experiment cannot operate at its maximum intensity if the large-scale exchange is accomplished by a meridional circulation. But, these are just the conditions under which the heat exchange in the experiment is observed to be accomplished mainly by asymmetrical disturbances rather than by a symmetrical meridional circulation. We are thus led to the hypothesis that the breakdown of symmetric flow when critical conditions are exceeded represents an attempt on the part of the general circulation to operate at its maximum intensity by introducing another type of exchange process when the symmetric type of exchange is insufficient. From the classical point of view, this breakdown is, of course, simply a manifestation of the instability of a symmetric baroclinic flow.

IV. Summary

In summary, we find that although only 0.02 of the incoming solar radiation is converted into kinetic energy in the atmosphere, this appears to be close to the maximum rate of conversion that is possible with the given distribution of incoming solar radiation. Thus, the general circulation is probably operating near its maximum possible intensity. Furthermore, it appears that at least in the case of the rotating fluid experiments, if the existing exchange process is too weak to maintain the general circulation at its maximum intensity, an alternate exchange process will be established which does maintain the general circulation at nearly its maximum intensity.

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